Exercise 03

1. Mathematics of Curvature

a) Consider the function $h(x_1, x_2) = x_1^2 + x_1 x_2 - 2x_2^2$ which we assume describes the shape of a deformed lipid bilayer membrane. x_1 and x_2 are the coodrinates of the reference plane below the membrane, as shown in figure 1. Draw a plot of the height as a function of x_1 and x_2.

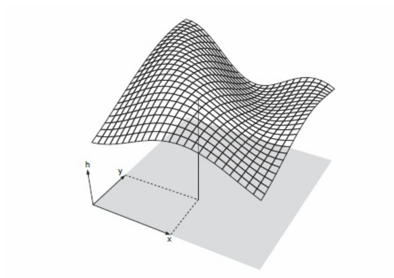

Figure 1: The height function, h(x, y). The surface of the membrane is characterised by a height at each point (x, y). This height function tells us how the membrane is disturbed locally from its preferred flat reference state.

b) Compute the principle radii of curvature as a function of x_1 and x_2.

c) Compute the bending free energy for the piece of membrane corresponding to the square $0 \leq x_1 \leq 1$ and $0 \leq x_2 \leq 1$ in the reference plane.

2. Distinguishable ligands

Derive the probability that a receptor is occupied by a ligand using a model which treats the L ligands in solution as distinguishable particles. Show that the expression is:

$$x = \frac{{}^{c}/_{c_0} e^{-\beta \Delta \epsilon}}{1 + {}^{c}/_{c_0} e^{-\beta \Delta \epsilon}}$$

where the ligands were treated as indistinguishable. c is the ligand concentration, c_0 a reference concentration corresponding to having all sites in the lattice occupied and $\Delta\epsilon = \epsilon_b - \epsilon_{sol}$, with ϵ_b the binding energy for the ligand and receptor and ϵ_{sol} the energy for the ligand in solution.